PILOT LEARNING
CALCULUS II ENGINEERING
PROBLEM-SET 5
FALL 2019

(1) Psychologists interested in learning theory study learning curves. A learning curve is the graph of a function P(t), the performance of someone learning a skill as a function of the training time t. The derivative dP/dt represents the rate at which performance improves.
   (a) Based on your own learning experience, sketch what you think a typical learning curve looks like.
      (i) When do you think P increases most rapidly?
      (ii) What happens to dP/dt as t increases?
   (b) If M is the maximum level of performance of which the learner is capable, explain why the differential equation

$$\frac{dP}{dt} = k(M - P)$$

   where k is a positive constant, is a reasonable model for learning.
   (c) Make a rough sketch of a possible solution of this differential equation. How does this graph compare to the one that you drew in (a)?

(2) Consider the differential equation $y' = x + y^2$.
   (a) Sketch the direction field of the differential equation.
   (b) Then use it to sketch a solution curve that passes through the point (0,0).

(3) Solve the following differential equations
   (a)

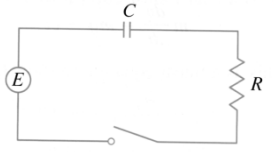
$$\frac{dv}{ds} = \frac{s+1}{sv+s}$$

   (b)

$$\frac{dy}{d\theta} = \frac{e^y \sin^2\theta}{y \sec\theta}$$

(4) Find an equation of the curve that passes through the point (0,1) and whose slope at (x, y) is xy.

(5) The figure below shows a circuit containing an electromotive force, a capacitor with a capacitance of C farads (F), and a resistor with a resistance of R ohms (Ω).



The voltage drop across the capacitor is Q/C, where Q is the charge (in coulombs). In this case, Kirchhoff's Law gives

$$RI + \frac{Q}{C} = E(t)$$

The rate at which the capacitor is charging equals I = dQ/dt, therefore,

$$R\frac{dQ}{dt} + \frac{Q}{C} = E(t)$$

(a) Suppose the resistance is 5 Ω, the capacitance is 0.05 F, a battery give a constant charge of 60 V, and the initial charge is Q(0) = 0 C. Find the charge and current at time t.

(b) Suppose the resistance is 2 Ω, the capacitance is 0.01 F, the initial charge is $Q(0) = 0$ C, and E(t) = 10 sin 60t. Find the charge and current at time t.